Vortex Wakes Behind High-Lift Wings

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At high wing lift coefficients pertinent to STOL operation, the conventional neglect of vortex roll-up effects can lead to errors when calculating downwash at the tail plane and in the presence of ground or wind-tunnel walls. A classical unsteady treatment in the cross-flow plane, which calculates the roll-up of an initial spanwise row of point vortices, has been modified to allow for the influence of the wing. Additional meaning is thereby given to the streamwise length dimension and hence to aspect ratio and sweep. The effects of height-above-ground and of various tunnel heights and widths are discussed. Under certain limited conditions, notably with part-span flaps or too narrow a tunnel, part or all of the trailing vortex system may move upwards. Consequent changes in the vertical velocity field are additional to conventional estimates involving only the appropriate image system.

I. Introduction

THE generation of high-lift coefficients for STOL operations is creating renewed interest in the phenomenon of vortex roll-up behind finite wings and in the subsequent vortex trajectory. This process can result in significant changes in local trailing vortex geometries relative to those assumed in conventional calculations, with consequent implications with regard to static stability. The vortex trajectory further downstream is relevant to nearby aircraft in operations such as in-flight refueling, closely spaced airport traffic, and air drops of cargo.

The results of calculations of vortex roll-up behind wings at high-lift coefficient will be presented, showing the effect of sweep, lift coefficient and incidence, the effect of approaching the ground, and the effect of wind-tunnel constraints for various cross sections.

The occurrence of vortex roll-up was recognized as early as 1907 by Lanchester[17] (see Fig. 1). There have been numerous attempts since then to describe the effect quantitatively. In 1935, Westwater[1] calculated vortex roll-up and trajectory using point, doubly infinite, vortex elements in a cross-flow plane and treating a two-dimensional unsteady problem in which time was equivalent to the mainstream direction. Betz[2] had already established some basic theorems. Since that time, numerous authors have investigated the subject both theoretically[3–5] and experimentally,[6] often by flight experiment.[7–10] The total problem, including axial flow in the vortex core[11] and viscous effects[12] was reviewed recently in Ref. 13.

The present method, unlike most others, takes into account the finite upstream length of the trailing vortices and adds the effect of the bound vortex system. Additional meaning is thereby given to the length dimension in the streamwise direction and consequently to wing aspect ratio and sweep.

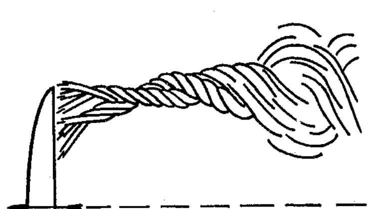

Fig. 1 Lanchester's drawing of vortex roll-up.

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Although the method is currently entirely nonviscous, viscous effects in the core and decay can be incorporated fairly readily.[16]

The numerical calculations were performed using a RAX remote-access 360–50 computer terminal at Lockheed-Georgia, and generally involved less than 10 min of computer time per run. The program has recently been modified to allow feedback of roll-up geometry into a vortex collocation program to determine the significance of roll-up effects on wing load distribution. A study is also anticipated of the effects of yaw on roll-up and consequent changes in cross flow at the tailplane.

Aerodynamic and numerical aspects of the methods used will be disucssed in Sec. II. The effects of varying some parameters, including height above the ground, will be examined in Sec. III. Wind-tunnel constraint will be discussed in Sec. IV, and Sec. V will deal with velocities induced by vortex systems previously determined.

II. Discussion of the Method

The two-dimensional calculation method[1] consists of the determination of velocity at each of the vortex points, due to

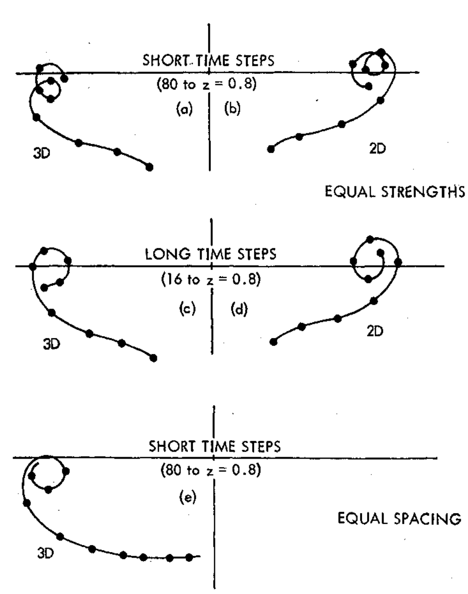


Fig. 2 Kinematic effects of step length, of quasi-three-dimensional treatment, and of type of vortex spacing.

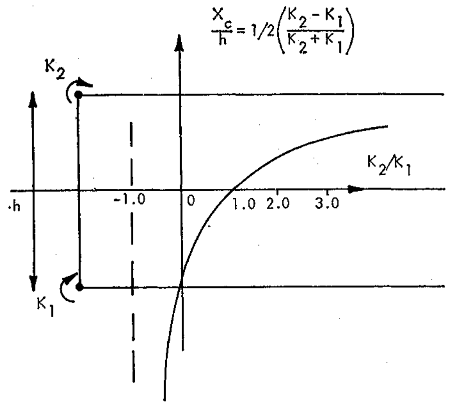


Fig. 3 a) Variation of position of common rotation point position with the relative strengths of vortices K_1 and K_2.

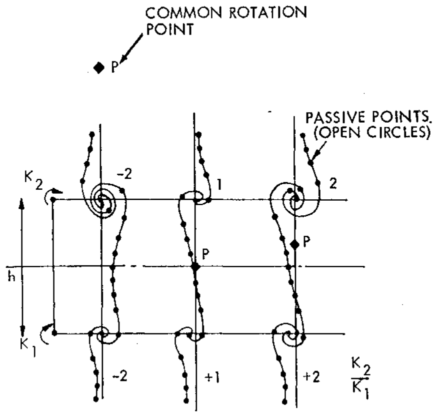


Fig. 3 b) Distortion of the initially straight line joining K_1 to K_2 (adapted from Fig. 11 of Ref. 14).

all other vortices, followed by the determination of their displacements in two dimensions during a finite time interval. An exact solution for vortex roll-up is reached only for a vanishing time interval. In practical cases a passive point, for example, moving under the influence of a potential vortex, experiences a radially outward displacement during a finite time step as well as the desired circumferential motion. Because of such effects, calculations involving the wake of a finite wing yield more diffuse rolled-up vortex groups for longer time steps than for smaller ones. This could be prevented by the use of a vortex function which induces a circular motion.[14] Although this actually is used in the present program when vortices approach too closely, it is too expensive in computer time. There are also strong theoretical objections to this procedure, since the implied velocity functions violate Laplace's Equation. An improved stepping method may be possible by taking into account information available from the previous one or two time steps. The chief benefit might be a saving in computer time brought about by the ability to use longer time steps. Turning now to actual examples, Figs. 2a and 2b shows that, although vortex grouping is tighter, as expected, for small time steps, the vortex sheet has crossed itself, which is physically impossible in real flow. In such situations, numerical stability problems caused by vortices approaching each other too closely can occur, and it is believed that better results may be achieved by using longer numbers in the calculation; 14-bit numbers were used for the calculations shown here.

There is ostensibly quite a wide choice of vortex arrangements along the trailing edge, ranging from equal strength vortices, spaced to give the right span loading, to equally spaced vortices of differing strengths. If one considers the motion of two vortices, strengths K_1 and K_2 (Fig. 3a) which are spaced a distance h apart, it may be shown that the center of rotation is a distance $[h/2][(K_2 - K_1)/(K_2 + K_1)]$ from the center point between the two vortices. As the relative strengths change, the center of rotation moves towards the stronger of two corotational vortices. The center of rotation no longer lies between vortices of opposite sense. Therefore, one might expect intuitively that the use of equal corotational vortices would be superior, but a comparison of Figs. 2a (equal

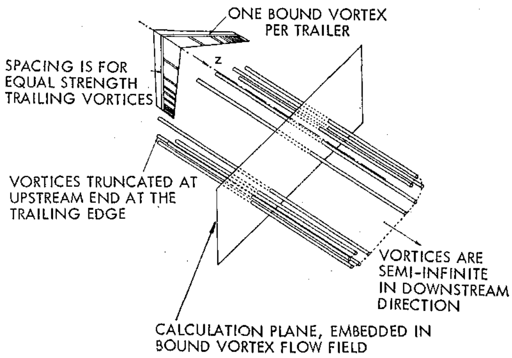


Fig. 4 Mathematical model of the roll-up process.

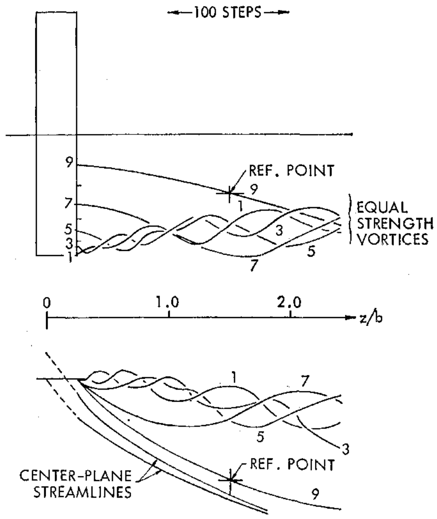


Fig. 5 Vortex roll-up in free air, C_L = 4.0, AR = 6.0 (equal strengths).

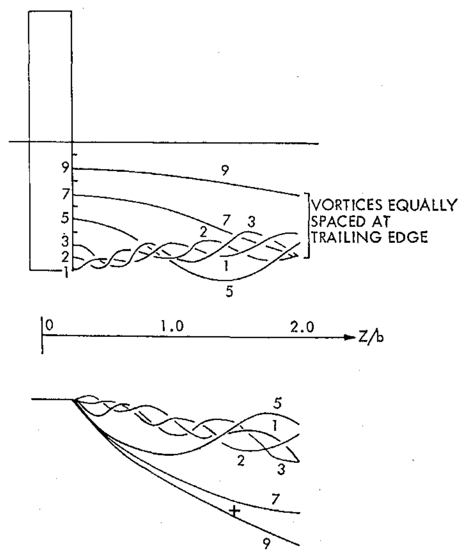


Fig. 6 Vortex roll-up in free air, C_L = 4.0, AR = 6.0 (equally spaced initially).

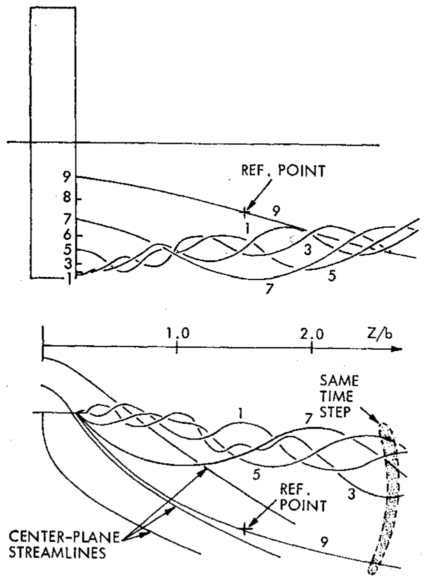


Fig. 7 Effect of allowing *Z* plane to distort (equal strengths).

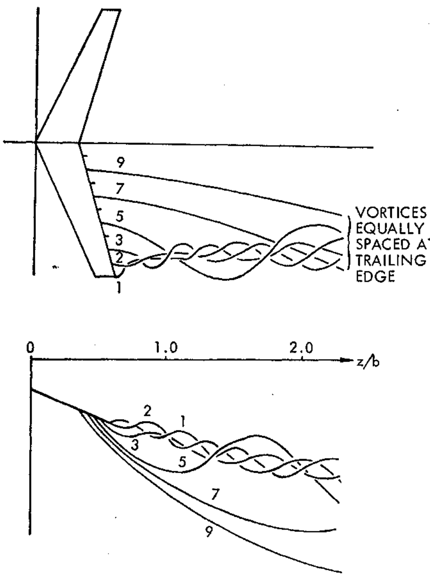


Fig. 9 Effect of wing sweep (vortex strengths are as for Fig. 6).

strengths) and 2e (equal spacing) shows the opposite to be true. The unequal spacings between several equal vortices seems to have influenced the issue, and a kinematic optimum between the two extremes might be expected. Figure 3b illustrates intervortex effects and shows the dangers of looking in too much detail near the singularities, a feature common to most finite mesh and singularity methods.

We turn now to a description of the quasi-three-dimensional approach. If a straight wing and a cross-flow plane situated at the trailing edge are considered, it is evident that the trailing part of the vortex system is initially semi-infinite and becomes double infinite only as the time plane approaches downstream infinity. One way of simulating this is to modify vortex strength in the cross-flow plane as a function of time. (Dissipative effects could also be included in this way.) At the trailing edge position, the bound and chordwise vortex systems in the wing have maximum effect on the wake, giving a distributed downwash velocity in the cross-flow plane. This diminishes and becomes more uniform as the calculation plane moves downstream. For wings with dihedral, or sweep-and-incidence, there are also bound-vortex-induced spanwise flow components. Figure 4 shows the flow model which is implied by the aforementioned treatment.

An example calculated by using the previous scheme for a straight wing of aspect ratio 6 at a lift coefficient of 4 is shown in Fig. 5. In this and in most of the subsequent examples, the detailed means of lift generation is unspecified. In Ref. 3 it is shown that the effect of differing chordwise distributions on downwash at the trailing edge is small. This has been confirmed in the present example by using two sets of bound vortices generating the same lift and moment; the resulting wake configuration was almost indistinguishable from Fig. 5. An important requirement in the present method is thus to simulate the geometry of the trailing edge correctly, particularly for swept wings and at incidence.

Before leaving Fig. 5 we note that, in contrast to earlier methods,[3] the center plane streamline of the vortex sheet is now slightly curved. Theorems due to Betz[2] probably no longer hold if spanwise velocity components are induced by the bound vortex system.

Figure 6 is for the same conditions as Fig. 5 but shows the effect of spacing the vortices equal distances apart. As noted in Fig. 2, this seems to produce a better organized vortex system, though the over-all effects are very similar.

While maintaining all of the features of the model shown in Fig. 4, the possibility of allowing out-of-plane distortions has also been considered. Figure 7 shows that the overall effect is not great, but as expected, the inboard vortices are retarded by the influence of the bound vortex once they move below quarter-chord height. For similar reasons, the outer layers of the roll-up, at a given time, are positioned slightly upstream of the core. Though it is interesting, the out-of-plane distortion seems to be justified only when center-plane streamlines are to be calculated, which can be done by putting appropriately situated zero-strength vortices into the calculation.

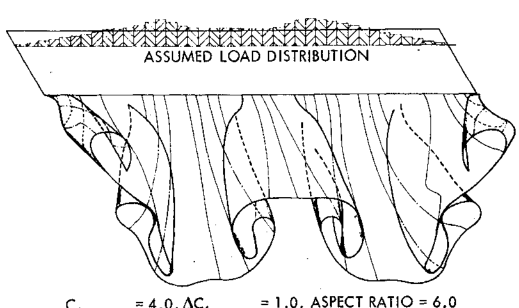


Fig. 8 Calculated vortex wake for a wing with partial-span flaps.

III. Further Examples, Including Ground Effect

Broadly speaking, the examples which will be given in this paper comprise variations on the basic straight wing case of Figs. 5 and 6, altering only one parameter at a time. Arrangements for the incorporation of flap trailing-edge geometry have not yet been made, but bound vortices skewed in three dimensions, as on a swept wing at incidence, can be accommodated.

With the exception of the swept-wing example, cases here have vortex sheets released from a trailing-edge position which is in the same horizontal plane as the bound vortex. It is expected that the main effect of incorporating geometric incidence would be to lower the whole sheet by an amount determined by the trailing-edge position. As for the fully two-dimensional vortex roll-up representation, variation in lift coefficient has its major effect on streamwise length scale measured from the trailing edge. Halving C_L, in Fig. 5

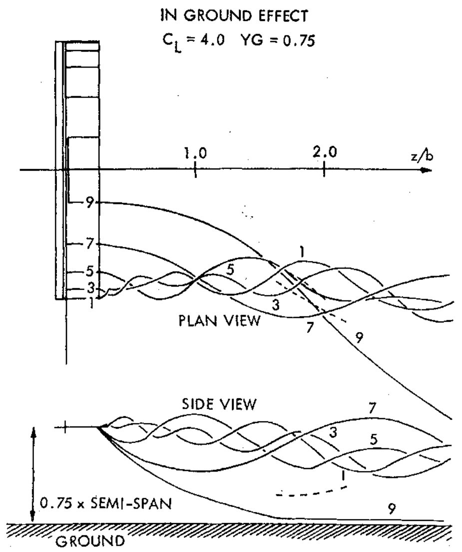


Fig. 10 C_L = 4.0, *AR* = 6, vortex roll-up in ground effect ¼-chord height = 0.75 × semispan (vortex strengths are as for Fig. 5).

for example, would produce a doubling of the distance to a particular cross-sectional pattern, though bound vortex effects would be incorrectly simulated.

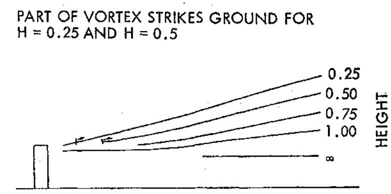
Figure 8 is an oblique view of the distorted vortex sheet calculated for an airfoil with part-span flaps. We see that secondary vortex pairs are formed, associated with the flaps, with the outermost flap vortices eventually becoming mingled with the tip vortices. By Z = 0.8 semispans, just beyond the last plane shown, the situation has become confused and better stepping arrangements, possibly with more vortex elements, seem desirable for better definition of the combination of the outermost pairs. Nevertheless, the patterns given here will probably yield acceptably good downwash patterns at the tailplane, for example.

The effects of wing sweep may be seen in Fig. 9, for which the span loading and vortex spacing are the same as for Fig. 6. Because of their earlier release, the innermost vortices already have a noticeable downward displacement (but a very similar trajectory in plan view) when the tip roll-up starts, and their incorporation into the rolled-up system is consequently delayed. In contrast, vortices at midsemispan seem to be fed into the tip roll-up earlier, probably as a result of a stronger local cross flow combined with the earlier release. Vortex number 5, for example, exhibits a steeper spiral for the swept case.

Ground effect may be simulated by the use of an image vortex system, reflected in the ground plane, which induces additional velocities during the roll-up process. Because of symmetry, the center of the wake would approach the ground asymptotically for infinitesimal time steps. However, for practical finite time steps, there is a real danger that a vortex will "penetrate" the ground and cause errors. It has proved possible to avoid this by providing a "buffer" layer next to the ground in which normal velocities are suppressed. Figure 10 shows that, when the innermost vortex reaches the ground at about Z = 1.75, the vertical motion is suppressed but the spanwise motion is uninterrupted.

Figure 10 also demonstrates the greater spanwise spacing of rolled-up vortices and a reduced downward motion in ground effect. The effect on vortex center distances is shown for greater downstream distances and other heights in Fig. 11. For economy of computer time it was necessary to use rather long time steps (corresponding to Fig. 2c), and cross-sectional distributions became somewhat dissipated. The mean lines shown describe the motion of the center of the vortex groups. It was observed that, for the two smallest heights, there was

Fig. 11 Effect of height above group upon vortex trajectory.



a pronounced flattening of the roll-up spiral in the vicinity of Z = 2. Further development was obscured by instabilities caused by the long step length and an insufficient number of vortices for these distances downstream. Mean effects, at least, probably could be studied better by using a single equivalent vortex pair. Dissipation effects should also be added, which would decrease the rate of spread further downstream.

IV. Wind-Tunnel Effects

Whereas the simulation of ground effect only doubles the number of vortices in the system, the image system for rectangular wind tunnels poses much greater problems. The classical doubly infinite sets of image vortex pairs obviously must be severely truncated for use in calculations at every time step.

In some early feasibility studies, ground, roof, and two wall image sets were used that seriously overestimated constraint phenomena. This led to a longer study, resulting in a compromise system shown in Fig. 12. The possibility of using a fully represented first "layer" and a modified second layer of images was studied, leading to results typified by Fig. 13. Here we see that, whereas the first image layer alone implies oval zero streamlines, two complete image layers imply a tunnel with slotted corners. Between these extremes a very good compromise, for the boundaries at least, is possible by attenuating vortex strength in the second layer by a factor F2. For points within the tunnel, a careful study[15] has indicated that a polynomial approximation in four variables may be used for F2 if required. This has been used to validate simpler approximations based on conditions at the tunnel boundary as in Fig. 13.

In Fig. 14, wake cross sections calculated using one image layer and using two image layers are presented for comparison with the free air shape shown in broken lines. As might be expected from the implied corner slots, the addition of a second layer generally shifts the rolling-up vortex downwards and outwards. The four tunnel cross sections shown are arranged so that tunnels b and c have $(2)^{1/2}$ times the width and height, respectively, of tunnel a. Tunnel d has both dimensions longer by the factor $(2)^{1/2}$. Thus, we may separate the effects of width, height, and size. We see that wall proximity causes tip vortices to sweep downward less and floor proximity has an obvious effect on the penetration of the center of the vortex sheet.

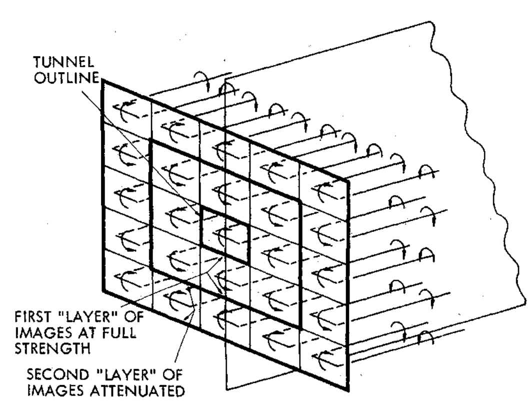


Fig. 12 Vortex image system used for tunnel representation during stepping process.

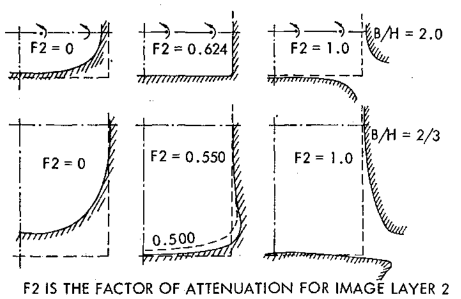


Fig. 13 Cross-flow zero streamlines for various systems of wind-tunnel wall images.

We may make some immediate comments on the implications of the roll-up patterns of Fig. 14 for unseparated flows. Broadly speaking, the size of tunnel d appears adequate from the point of view of wake distortion, though direct effects on loading and load distribution have not been studied. Slightly increased downwash at the tail may be expected, due to lessened sheet penetration, and some more serious implications for the yawed condition, particularly at high incidence. Though the yawed wake calculations have not yet been attempted, it may be inferred that, as pitch is increased in this elliptically loaded case, a wind-tunnel model will experience a destabilizing yawing moment due to trailing vortex action sooner than the corresponding flight case in free air, due to the decreased downsweep of vortices in the wind tunnel. As tunnel width is decreased (cases a and c), the effect will worsen. Further work on yawed cases is planned. particularly where secondary roll-up systems are present, such as in Fig. 8. Here, further calculations must precede any discussion about stability effects. It seems, however, that care will be needed in predicting lateral stability from wind-tunnel tests because of wake distortion. The variation of static yaw derivatives with pitch angle may be affected, and probably also the dynamic ones.

It is also possible to make some limited inferences from Fig. 14 and similar calculations concerning deep stall. We may expect that if the depth of the separated wake is not too large, the behavior of the thick wake will be similarly affected. Thus, the horizontal surfaces of a "*T*" empennage would be blanked by wake air at a lower incidence in the wind tunnel than in free flight. This might give a more demanding criterion for choice of tunnel size or shape than the lateral considerations.

V. Streamlines and Induced Velocities

The vortex paths calculated for Figs. 5 and 6, for example, would be streamlines if out-of-plane distortions of the time

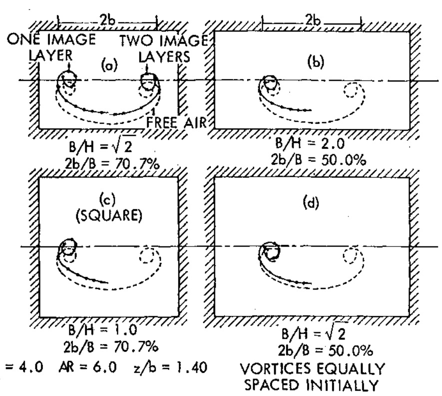


Fig. 14 Effect of tunnel size and shape on vortex roll-up and trajectory.

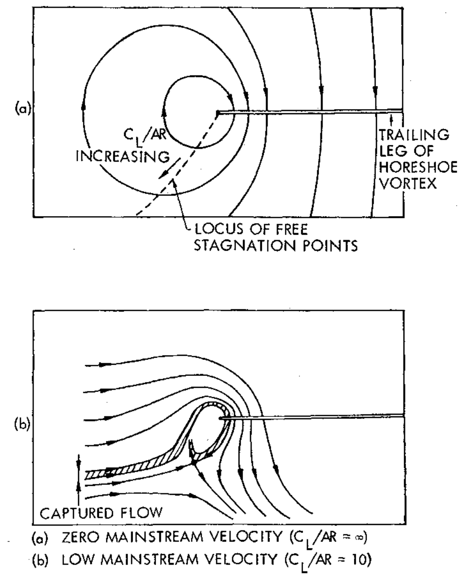


Fig. 15 Three-dimensional streamlines in the vertical center plane for a horseshoe vortex in free air (taken from Ref. 15).

plane had been allowed. Figure 7 showed that allowing this freedom did not greatly affect the roll-up geometry.

By putting vortices having zero strength into the vortex roll-up program and allowing this streamwise distortion once again, we may examine the effect of ground or wind-tunnel constraint on streamlines in the vicinity of the tail-plane, which is likely to become particularly relevant in ground effect and under the influence of wind-tunnel constraint. First, however, some comments on the nature of three-dimensional streamlines will provide a helpful perspective.

In Fig. 15a, streamlines due to an isolated horseshoe vortex are shown, with no freestream present. The familiar two-dimensional concentric streamlines are replaced by a system with centers offset away from the trailing vortex legs. Adding a freestream (Fig. 15b) gives rise to a free stagnation point, as in two dimensions. In this three-dimensional case, however, a spiral vortex is formed which "captures" a segment of freestream air which is convected outwards along the span from planes on each side of the center. We note that streamline spacing no longer is a reliable indicator of local velocity. (Further examples, including ground and wind-tunnel effects, may be found in Ref. 15.)

Figures 16 and 17 show streamlines in the center plane for a ground-effect and a wind-tunnel case, respectively. Unless otherwise stated, these are calculated for wakes which are rolling up. However, to demonstrate the changes due to roll-up, streamlines for a horizontal rigid wake have been added in Fig. 16, using the same basic vortices. We see that local effects from the most inboard vortex distort the streamlines and that if a rigid vortex system must be used, a single horseshoe vortex representing the wing gives better results. Never-

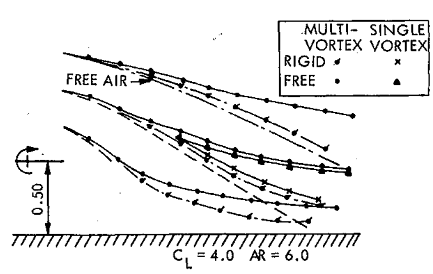


Fig. 16 Streamlines in the longitudinal center plane.

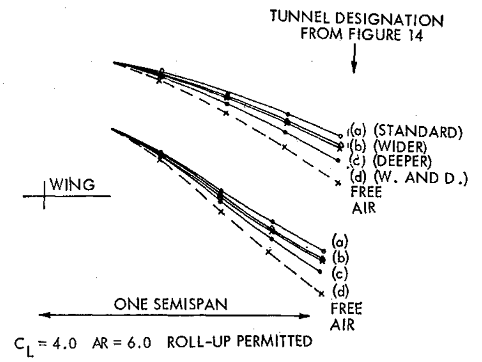


Fig. 17 Streamlines in the longitudinal center plane for various tunnel sizes and shapes.

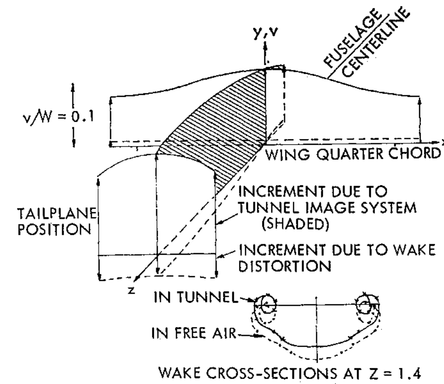


Fig. 18 Upwash increments induced by ground constraint effects.

theless, the separation of roll-up effects from the basic ground or tunnel image-induced effects by consideration of streamlines is conjectual. The chief virtue in the present context is in showing pitch angles in the flow.

The full lines in Figs. 18 and 19 show incremental upwash velocities, induced by ground and wind-tunnel constraints, respectively, and include the effects of wake distortion. The roll-up cross-sections in these figures (as in Fig. 14) show a flattening of the center of the wake due to the ground or tunnel floor. The duplex tunnel is clearly not high enough, relative to span, and upwash near the center is similar to that for ground effect but greater because of the roof. At the tailplane, the total upwash increments for ground and tunnel effects are similar; this may be due partly to the diagonal tunnel images and consequently may depend on tunnel geometry.

The broken lines in Figs. 18 and 19 show velocity increments induced by wake distortion, which are included in the totals just discussed. The roll-up details in these figures show an outward motion of the tip vortices in ground effect which is absent in the tunnel. The geometry near the center of the wake is very similar for both cases: vortices move upwards noticeably more than they do outwards. There are thus two opposite tendencies present: outward tip motion reduces downwash at the tail whereas a predominantly upward motion of the innermost vortices increases it. The former effect dominates for the ground effect case, giving a resultant upwash increment due to wake distortion in ground effect, whereas the opposite is true for the particular model and wind-tunnel combination of Fig. 19. We see that interference increments at the tailplane derive from two opposite effects which separately can be of important magnitude. It is not generally possible therefore to predict the sense of the interference due to wake distortion even though it may be of important size.

In all of the aforementioned we have used the same spanwise load distributions and intensities to identify effects due solely to distortion of the wake. The over-all picture when loadings are modified appropriately for ground or tunnel constraint is to be the subject of further investigations.

The detailed nature of upwash interference due to wake distortion also requires further investigation, particularly to ensure that the central part of the vortex sheet is represented properly and that errors due to the use of point vortices are minimized.

VI. Conclusions

A method for calculating vortex roll-up behind finite wings has been described which relies upon the perturbation of semi-infinite, finite-strength vortices embedded in a three-dimensional velocity field. Using this method, it is possible to carry out computer experiments which are analogous to flow visualization experiments and to study the effects of changes in wing geometry or spanwise load distribution and the effects of ground or wind-tunnel wall constraints. Sets of streamlines can also be added, using a fully three-dimensional perturbation method, in the same way that a smoke rake can be used in a wind tunnel.

Studies have been made, for the high-lift coefficients appropriate to STOL, which show the effect of vortex roll-up and on wake shape of making changes in geometry or imposing constraints as mentioned above. These are in agreement with general experience and follow the expected trends. However, for various nontechnical reasons, it has not been possible to carry out the necessary experimental checks on the validity of the method; data are needed on the vortex roll-up behind a simple wing generating high lift (preferably without auxiliary blowing) having a known spanwise load distribution. Therefore, the quantitative results presented here, notably velocity distributions, must be considered tentative.

Nevertheless, some interesting conclusions are possible concerning the changes in vertical velocity increments which are implied by calculated wake distortions due to ground or wind-tunnel constraints. When the wake is distorted by these constraints, two opposite effects contribute to vertical induced velocities which may individually be of important size, particularly at the tailplane. Outward motion of the tip vortices, induced by ground or tunnel floor/roof images, tends to reduce the downwash at the tail. Upward distortion of the center of the wake tends to increase this downwash. For the limited cases studied so far, such distortion effects increased the upwash increment due to ground constraint, but gave a downward (distortion-induced) increment in the wind tunnel, thereby reducing the total tunnel-induced upwash.

The implications of these changes on longitudinal static (and probably dynamic) stability could well be important.

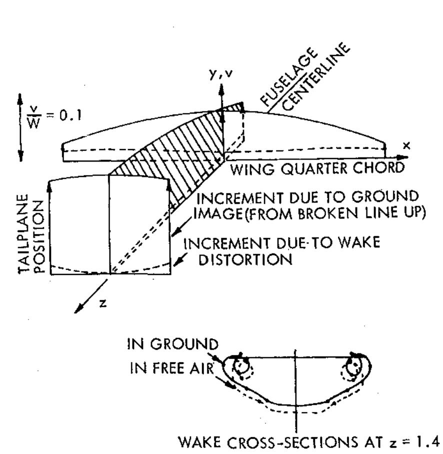


Fig. 19 Upwash increments induced by wind-tunnel constraint effects.

The generally higher positions of the trailing vortices near the ground or in the wind tunnel may need to be taken into account when considering both longitudinal and lateral stability and control during STOL.

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